Advanced composites in engineering structures

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Lecture #5

Strength of composite materials

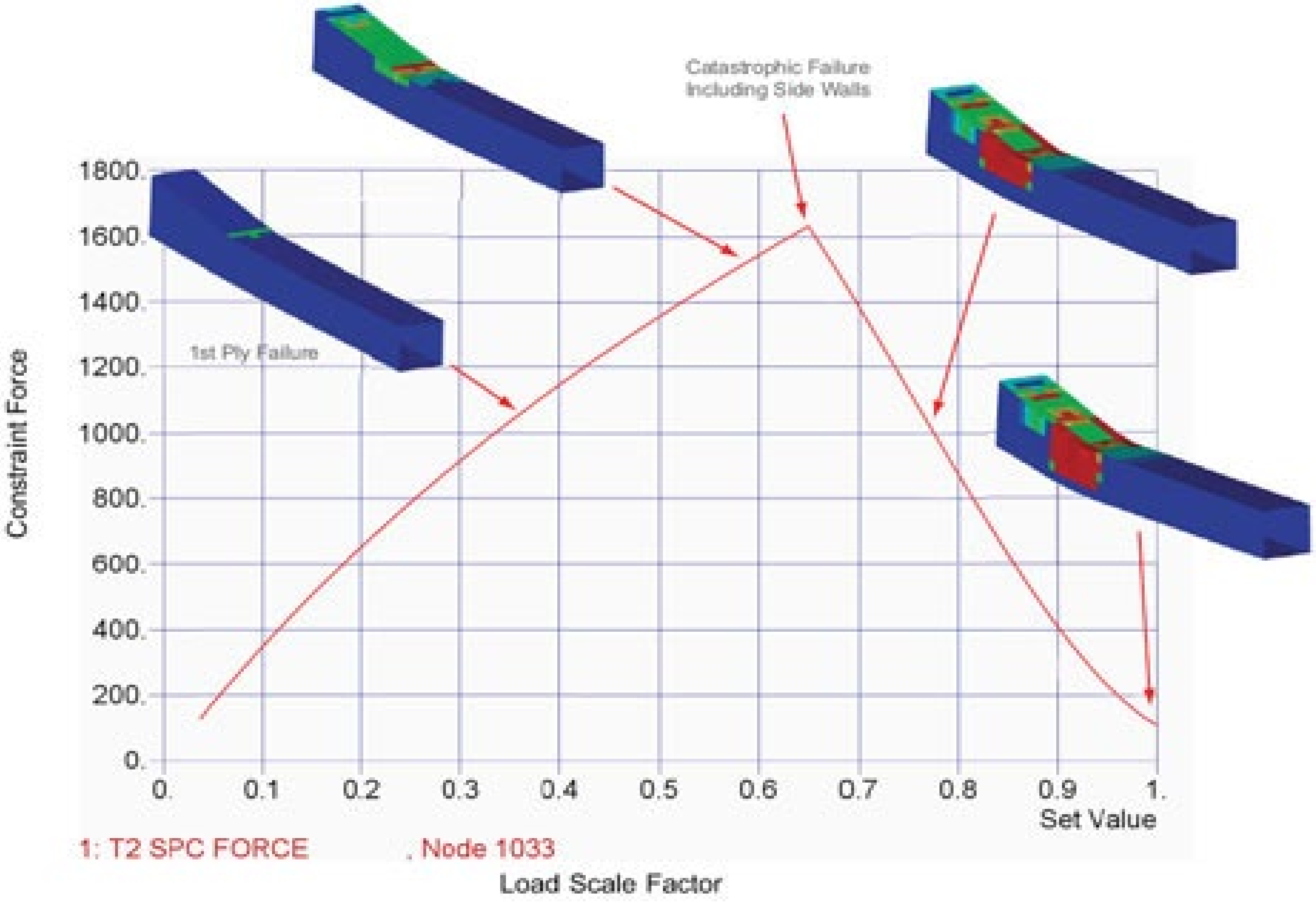
Strength of (composite) materials

- For composite materials:
  - The “one-constant” approach for strength or for stiffness is (usually) no longer adequate
  - (For the simplest case of the unidirectional laminate) four elastic constants are needed for the stiffness and six constants for the strength.
  - Unidirectional composites have highly directionally dependent strengths. Therefore, for any state of applied stress, all stress components must be examined before judgment on the cause of failure can be made.

Failure criteria

- For the analysis of composite laminates (consisting of a number of unidirectional plies) we need a failure criterion for the unidirectional layer.

- We would expect successive ply failures as the applied load to a laminate increases. The First Ply Failure (FPF) is followed by other ply failures until the Last Ply Failure (LPF) which would be the ultimate failure of the laminate.



Catastrophic Failure
Including Side Walls
1st Ply Failure
Constraint Force
1800.
1600.
1400.
1200.
1000.
800.
600.
400.
200.
0.
0.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.
Set Value
1: T2 SPC FORCE
, Node 1033
Load Scale Factor

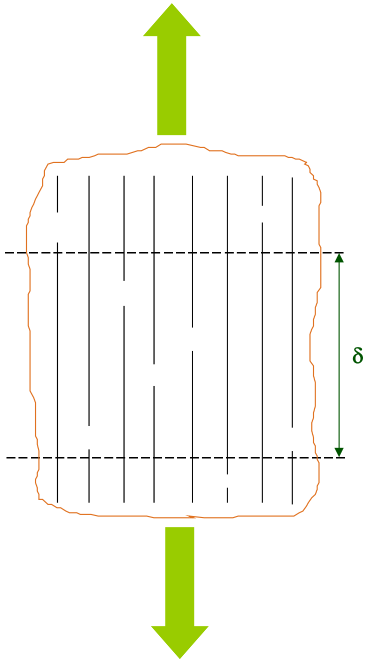
Composite failure

- Composites fail in a different way compared to metals
  - They exhibit different failure modes
  - They have brittle fibers in less brittle, occasionally ductile matrices

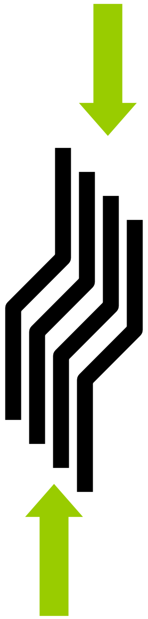
    Therefore, we can see

    - Sudden failures
    - Crazing and matrix cracking
    - Delamination and debonding
    - Usually unseen damage from defects/voids etc initiates in the laminate

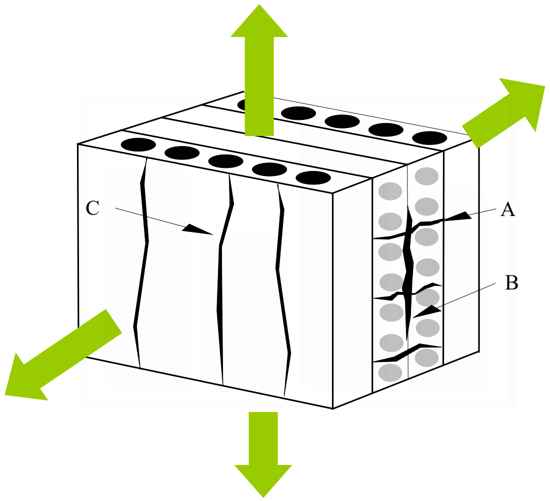
Failure Modes



Tensile Failure



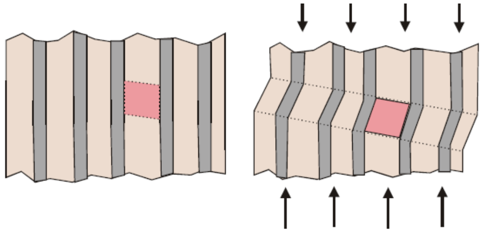
Compression Failure



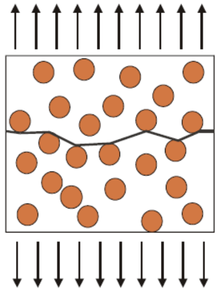
Matrix Cracking
& Delamination

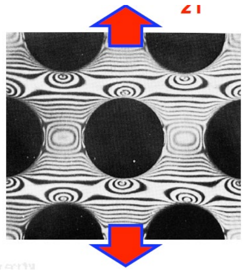
Loadings

Axial compression

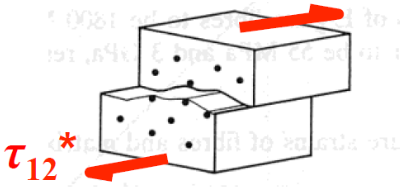


Transverse tension



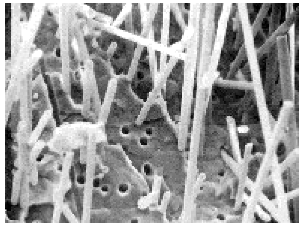


Axial shear

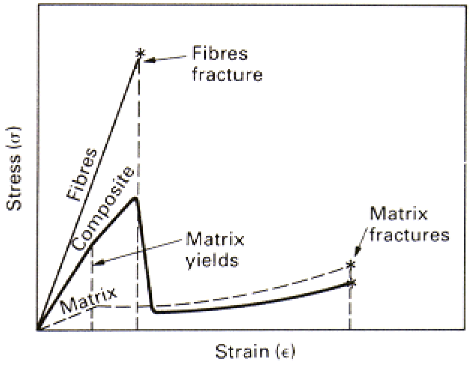


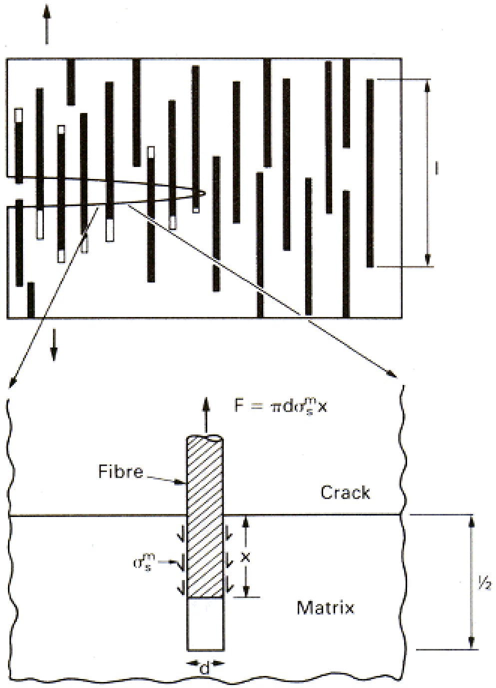
Failure at the micro- and macroscale

Fiber pull-out & fiber failure

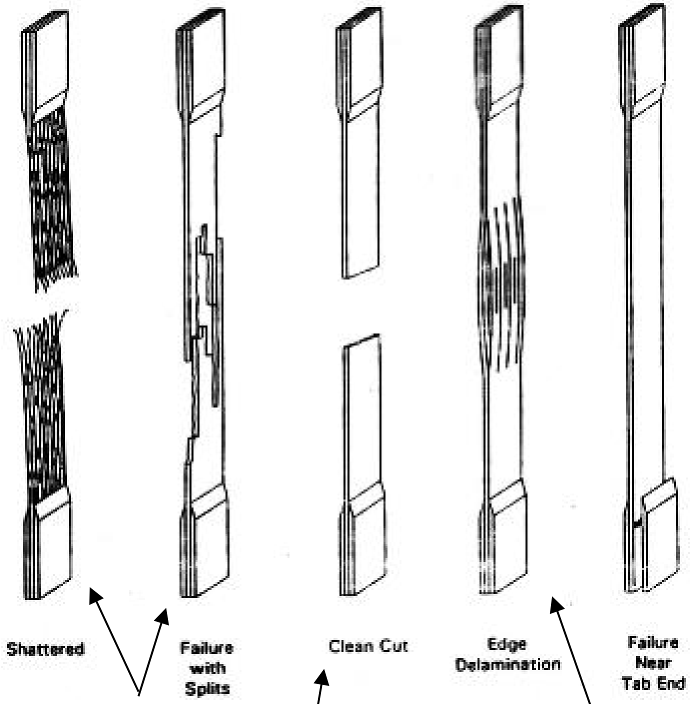


Stress strain curve of a composite





Typical Failure Modes in Straight-Sided Coupons



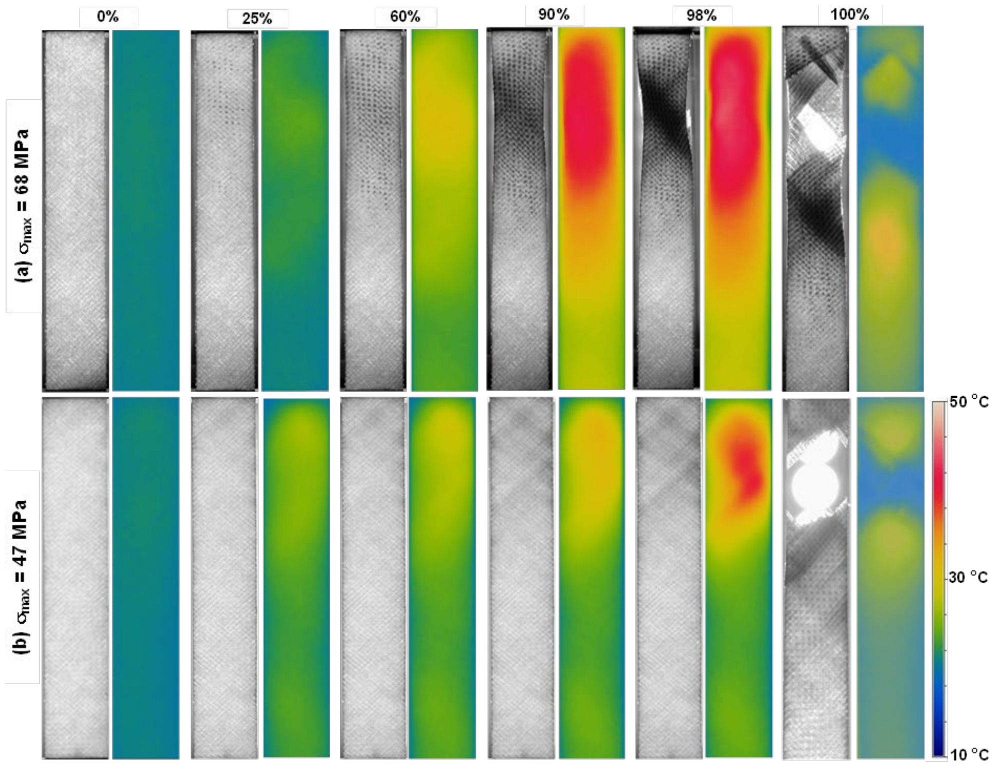
(Unacceptable)

(Acceptable & common in unidirectional specimens)

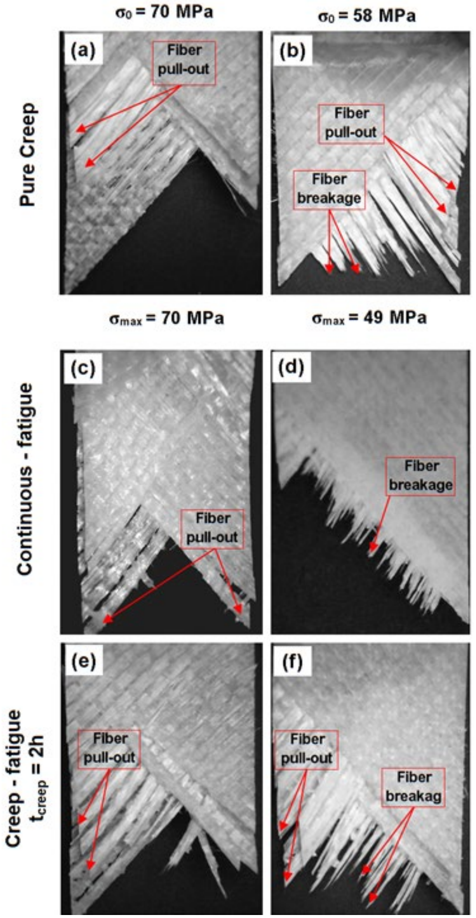
(Acceptable & common in 90˚ or 90˚ dominated layups)

(May be found in cross-ply layups; unacceptable)

Damage monitoring



Optical microscopy



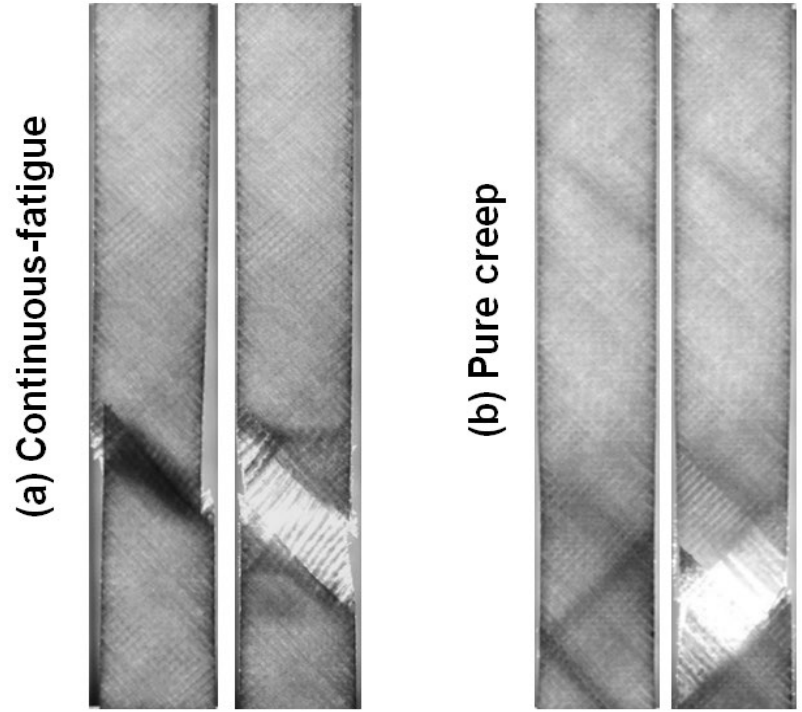
Lateral view of

90^{o} off-axis specimen

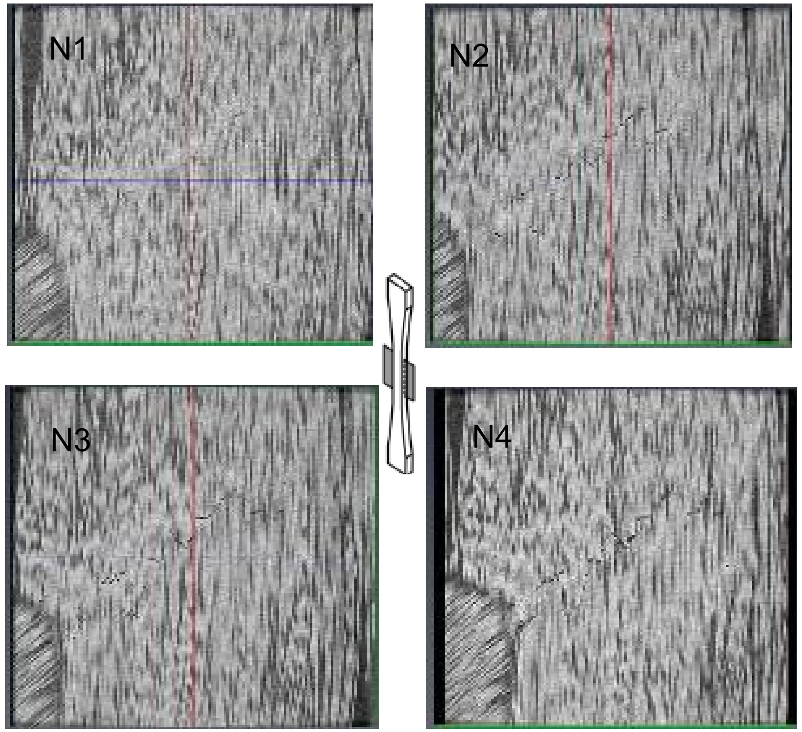
under R = –1 at 38 MPa

Damage monitoring

Optical inspection



Magnetic tomography



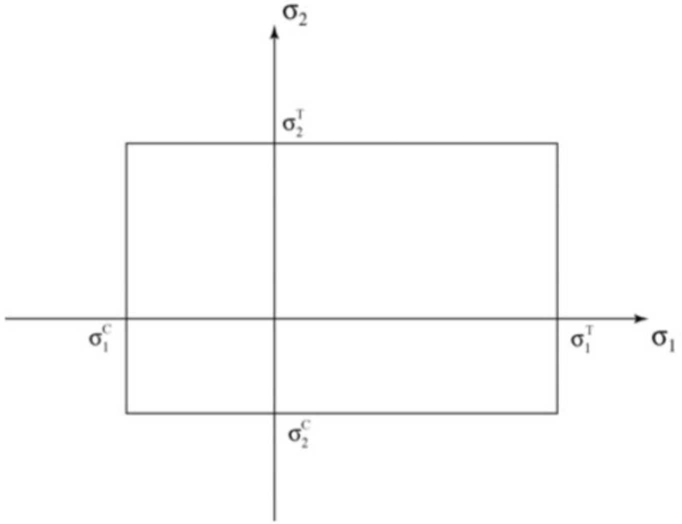
Two main approaches for failure analysis

- a) Maximum stress and strain criteria
  - Not interactive failure criteria associated with failure modes
- b) Quadratic interaction criteria
  - Interactive failure criteria that can be associated or not with failure modes

Maximum stress/strain

Related to the maximum normal stress theory by Rankine and the maximum shear stress theory by Tresca, this theory is similar to those applied to isotropic materials.

The stresses acting on a **lamina** are resolved into the normal and shear stresses in the **local axes**. Failure is predicted in a lamina, if any of the stress components is equal to or exceeds the corresponding ultimate strengths of the material



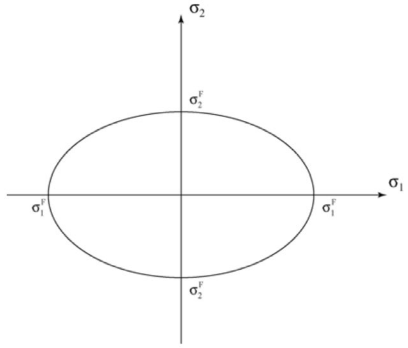
$$-X' < \sigma_1 < X$$
$$-Y' < \sigma_2 < Y$$
$$-S < \sigma_6 < S$$

OR for the maximum strain theory:

$$-\left(\varepsilon_1^c\right)_{ult} < \varepsilon_1 < \varepsilon_{1ult}^t$$
$$-\left(\varepsilon_2^c\right)_{ult} < \varepsilon_2 < \varepsilon_{2ult}^t$$
$$-\left(\varepsilon_6\right)_{ult} < \varepsilon_6 < \varepsilon_{6ult}$$

For both cases: Each component of stress (strain) has its own criterion and it is not affected by the other components, i.e., **there is no interaction**

Quadratic interaction criteria



- $F_{ij}\sigma_i\sigma_j+F_i\sigma_i=1$

- And for the strain components:
  - $G_{ij}\varepsilon_i\varepsilon_j+G_i\varepsilon_i=1$
- Where the *F*'s and *G*'s are strength parameters analogous to the constants in equations of maximum stress/strain criteria. Failure occurs when equation is met.

Quadratic interaction criteria

Layer!

– $G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1$

$$G_{11}\varepsilon_1^2 + 2G_{12}\varepsilon_1\varepsilon_2 + G_{22}\varepsilon_2^2 + G_{66}\varepsilon_6^2 + G_1\varepsilon_1 + G_2\varepsilon_2 = 1$$

Where:

$$G_{11} = F_{11}Q_{11}^2 + 2F_{12}Q_{11}Q_{12} + F_{22}Q_{12}^2$$

$$G_{22} = F_{11}Q_{12}^2 + 2F_{12}Q_{11}Q_{12} + F_{22}Q_{22}^2$$

$$G_{66} = F_{11}Q_{11}Q_{12} + F_{12}\left(Q_{11}Q_{22} + Q_{12}^2\right) + F_{22}Q_{12}Q_{22}$$

$$G_1 = F_1Q_{11} + F_2Q_{12}$$

$$G_2 = F_1Q_{12} + F_2Q_{22}$$

Use of failure criteria

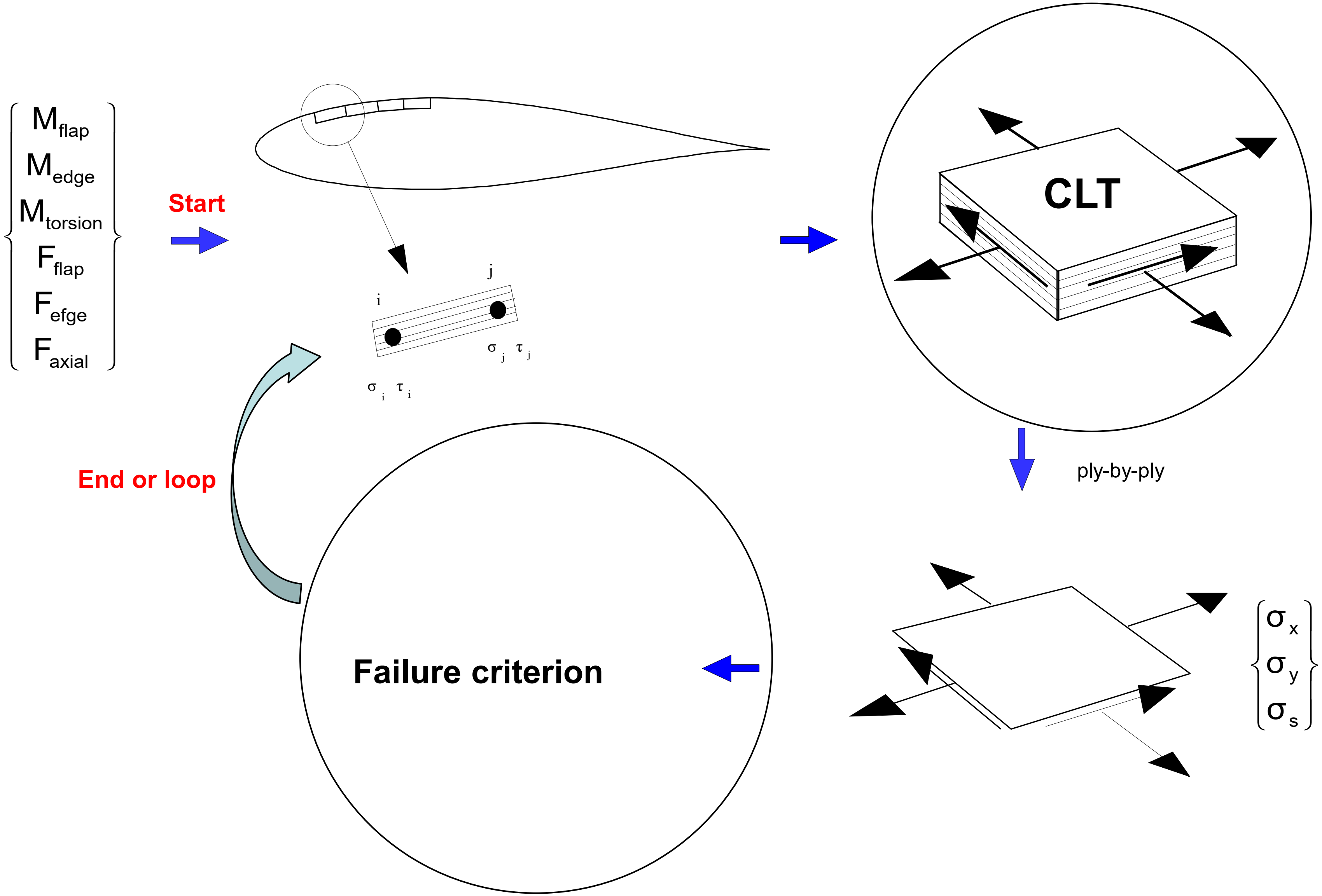
- Failure criteria constitute the analytical mathematical expressions for the calculation of the failure under complex loading conditions as a function of the “basic” material properties, that can be determined through “simple” tests.

- Failure criteria serve important functions in the design and sizing of composite laminates. **However, the criteria are not intended to explain the mechanics of failure (although…).** The framework should remain the same for different definitions of failure, such as the ultimate strength, the yield point the endurance limit or even a working stress based on reliability considerations.

- **The quadratic criterion will be analyzed here**. It is simple and versatile. It considers interaction between the stress or strain components analogous to the von Mises criterion for isotropic materials.

Implementation of failure criteria in a design cycle

Or - How and where do we use a failure criterion



Tsai-Hill

Criterion that has been proposed by R. Hill* for orthotropic media. It is a generalization of the von-Mises distortional energy yield criterion for isotropic materials.

$$F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L\sigma_4^2 + 2M\sigma_5^2 + 2N\sigma_6^2 = 1$$

With:

$$2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\ ,\ 2H = \frac{1}{Y^2} + \frac{1}{X^2} - \frac{1}{Z^2}\ ,\ 2G = \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2}$$

$$2L = \frac{1}{R^2}\ ,\quad 2M = \frac{1}{T^2}\ ,\quad 2N = \frac{1}{S^2}$$

X, Y, Z, R, T, S: the material strength under normal and shear loads

Tsai-Hill criterion does not consider for the different strength of the anisotropic material under tensile or compressive loads or positive-negative shear…

*R. Hill, "A Theory of the Yielding and Plastic Flow of Anisotropic Metals" Proc. Royal Soc. A193, pp.281-297 (1948)

Substituting, Hill's criterion can be written as: (on the local system):

$$\frac{\sigma_1^2}{X^2}+\frac{\sigma_2^2}{Y^2}+\cancelto{0}{\frac{\sigma_3^2}{Z^2}}-\left(\frac{1}{Y^2}+\frac{1}{Z^2}-\frac{1}{X^2}\right)\cancelto{0}{\sigma_2\sigma_3}-\left(\frac{1}{X^2}+\frac{1}{Z^2}-\frac{1}{Y^2}\right)\cancelto{0}{\sigma_1\sigma_3}-\left(\frac{1}{Y^2}+\frac{1}{X^2}-\frac{1}{Z^2}\right)\sigma_1\sigma_2+\cancelto{0}{\frac{\sigma_4^2}{R^2}}+\cancelto{0}{\frac{\sigma_5^2}{T^2}}+\frac{\sigma_6^2}{S^2}=1$$

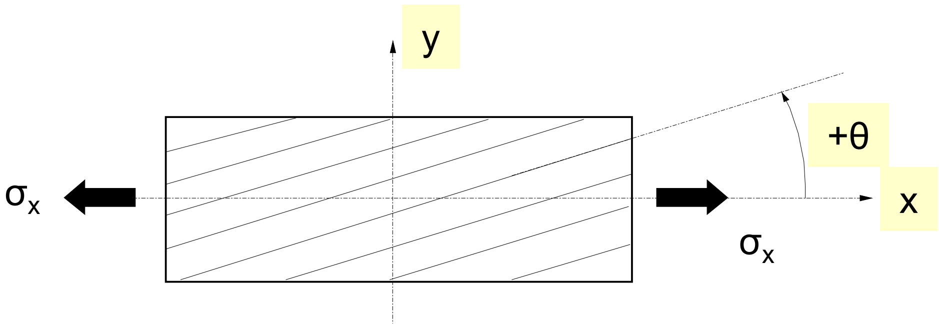
And for plane stress (plane 1-2):

$$\frac{\sigma_1^2}{X^2}+\frac{\sigma_2^2}{Y^2}-\left(\cancel{\frac{1}{Y^2}}+\frac{1}{X^2}-\cancel{\frac{1}{Z^2}}\right)\sigma_1\sigma_2+\frac{\sigma_6^2}{S^2}=1$$

And for a transversely isotropic medium (UD):

$$\frac{\sigma_1^2}{X^2}+\frac{\sigma_2^2}{Y^2}-\frac{\sigma_1\sigma_2}{X^2}+\frac{\sigma_6^2}{S^2}=1$$

Example: Prediction of failure of an off-axis lamina under uniaxial tensile load (prediction of the off-axis strength!)

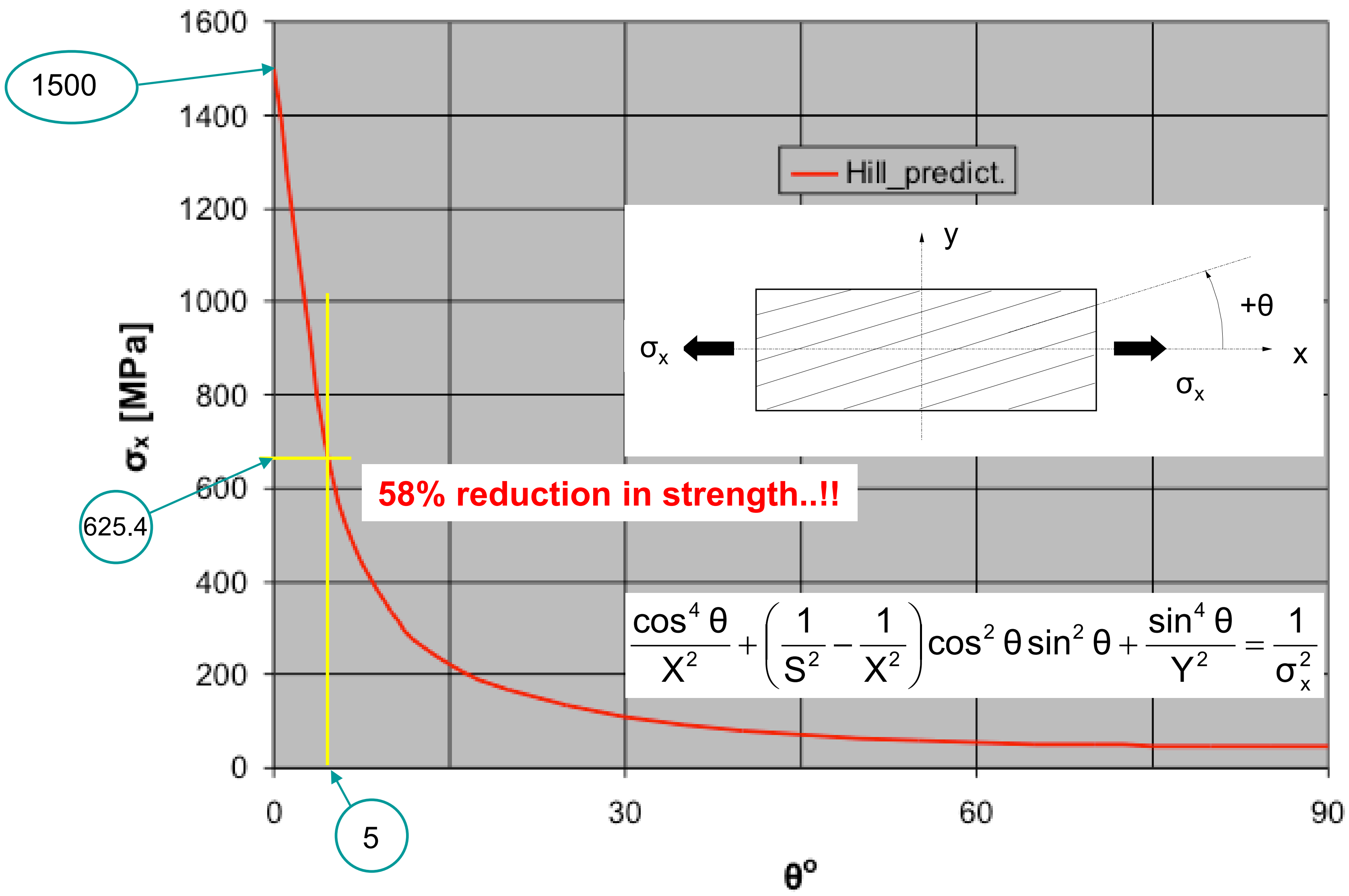


$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{Bmatrix}=\begin{bmatrix}m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & m^2-n^2\end{bmatrix}\begin{Bmatrix}\sigma_x\\ 0\\ 0\end{Bmatrix}$$

m=cosθ, n=sinθ

$$\boxed{\frac{\cos^4\theta}{X^2}+\left(\frac{1}{S^2}-\frac{1}{X^2}\right)\cos^2\theta\sin^2\theta+\frac{\sin^4\theta}{Y^2}=\frac{1}{\sigma_x^2}}$$

Off-axis strength of UD FRP



Failure tensor polynomial

For orthotropic media, in the local system:

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

There exist 12 independent failure components that should be determined through tests

If the stresses are known for another co-ordination system then:

$$F'_{11}\sigma_1'^2 + F'_{22}\sigma_2'^2 + F'_{33}\sigma_3'^2 + F'_{44}\sigma_4'^2 + F'_{55}\sigma_5'^2 + F'_{66}\sigma_6'^2 +$$
$$2(F'_{12}\sigma'_1\sigma'_2 + F'_{13}\sigma'_1\sigma'_3 + F'_{23}\sigma'_2\sigma'_3) + F'_1\sigma'_1 + F'_2\sigma'_2 + F'_3\sigma'_3 - 1 = 0$$

F'_{ij} unknown, but functions of F_{ij} !

It is always preferable to calculate the stresses for the local system, where the failure components are known.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma'_1 \\ \sigma'_2 \\ \sigma'_6 \end{Bmatrix} \longrightarrow$$

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

m=cosθ, n=sinθ

Determining the failure components

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(F_{12}\sigma_1\sigma_2 + F_{13}\sigma_1\sigma_3 + F_{23}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0$$

Tensile test in 1-direction

$$\sigma_1 \neq 0,\ \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = \sigma_6 = 0$$

$$F_{11}\sigma_1^2 + F_1\sigma_1 = 1$$ At failure σ_1=X $$F_{11}X^2 + F_1 X = 1$$

Compression in 1-direction

$$F_{11}\sigma_1^2 + F_1\sigma_1 = 1$$ At failure σ_1=-X' $$F_{11}X'^2 - F_1 X' = 1$$

$$\left.\begin{array}{l} F_{11}X^2 + F_1 X = 1 \\ F_{11}X'^2 - F_1 X' = 1 \end{array}\right\} F_{11} = \frac{1}{XX'} \quad , \quad F_1 = \frac{1}{X} - \frac{1}{X'}$$

accordingly:

$$F_{22} = \frac{1}{YY'} \quad , \quad F_2 = \frac{1}{Y} - \frac{1}{Y'} \qquad F_{33} = \frac{1}{ZZ'} \quad , \quad F_3 = \frac{1}{Z} - \frac{1}{Z'}$$

Pure shear test on 1-2 plane

For the orthotropic medium the shear strength is not dependent on the sign:

$$R^+ = R^- \quad , \quad T^+ = T^- \quad , \quad S^+ = S^- \longrightarrow F_{44} = \frac{1}{R^2} \quad , \quad F_{55} = \frac{1}{T^2} , \quad F_{66} = \frac{1}{S^2}$$

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 +$$
$$2(\boxed{F_{12}}\sigma_1\sigma_2 + \boxed{F_{13}}\sigma_1\sigma_3 + \boxed{F_{23}}\sigma_2\sigma_3) + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 - 1 = 0 \qquad F_{ij},\ i \neq j : stress\ interaction\ terms$$

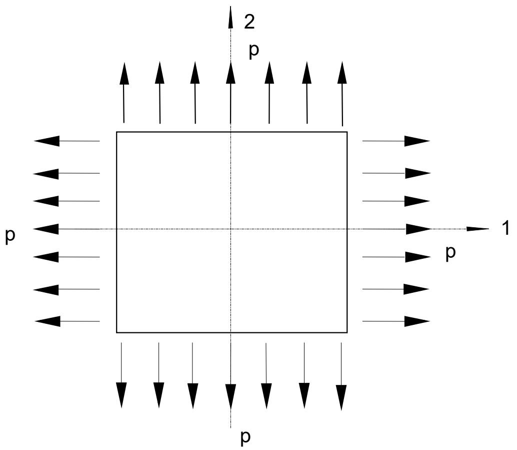
With 6 simple tests and 3 “almost” simple the failure components can be measured experimentally.

What can be done with the complex terms, that designate the stress interaction?

Determination of the stress interaction terms leads to different criteria!!!

A. Tsai-Wu[1]

Determine F_{ij} through biaxial tests:



$$(F_{11} + F_{22})p^2 + 2F_{12}p^2 + (F_1 + F_2)p - 1 = 0$$

$$F_{12} = \frac{1}{2p^2}\left\{1 - (F_{11} + F_{22})p^2 - (F_1 + F_2)p\right\}$$

Apparently, same F_{12} value should be measured for different loading, e.g., σ_1=p and σ_2=3p

1. S. W. Tsai and E. M. Wu: A General Theory of Strength for Anisotropic Materials, AFML-TR-71-12 (1971)

B. Tsai-Hahn[1]

Theoretical determination of F_{ij} terms through the equations:

$$F_{ij} = -\frac{1}{2}\sqrt{F_{ii}F_{jj}} \qquad i,j=1,\ldots,3 \qquad i \neq j \qquad e.g.\ \ F_{12} = -\frac{1}{2}\sqrt{F_{11}F_{22}}$$

C. Narayanaswami-Adelman[2]

$$F_{ij} = 0 \qquad i,j=1,\ldots,3 \qquad i \neq j$$

F_{ij} should be neglected because of their low values. Good for in-plane off-axis uniaxial laoding. Problems occur when complex plane stress field exists or for 3D stress fields.

D. Wu-Stachurski[1]

$$F_{ij} = \frac{-F_{ii}F_{jj}}{F_{ii}+F_{jj}} \qquad i,j=1,\ldots,3 \qquad i \neq j$$

Empirical, based on experimental data on specific material.

E. Hoffman[3]-EPFS[4]

$$F_{ij} = \frac{1}{2}\left(F_{kk} - F_{ii} - F_{jj}\right) \qquad i,j,k=1,\ldots,3 \qquad i \neq j \neq k$$

1. R. Y. Wu and Z. Stachurski, J COMPOS MATER 18, p.456 (1984)

Experimental evidence:

In certain cases FTP provides more accurate results compared to other failure criteria

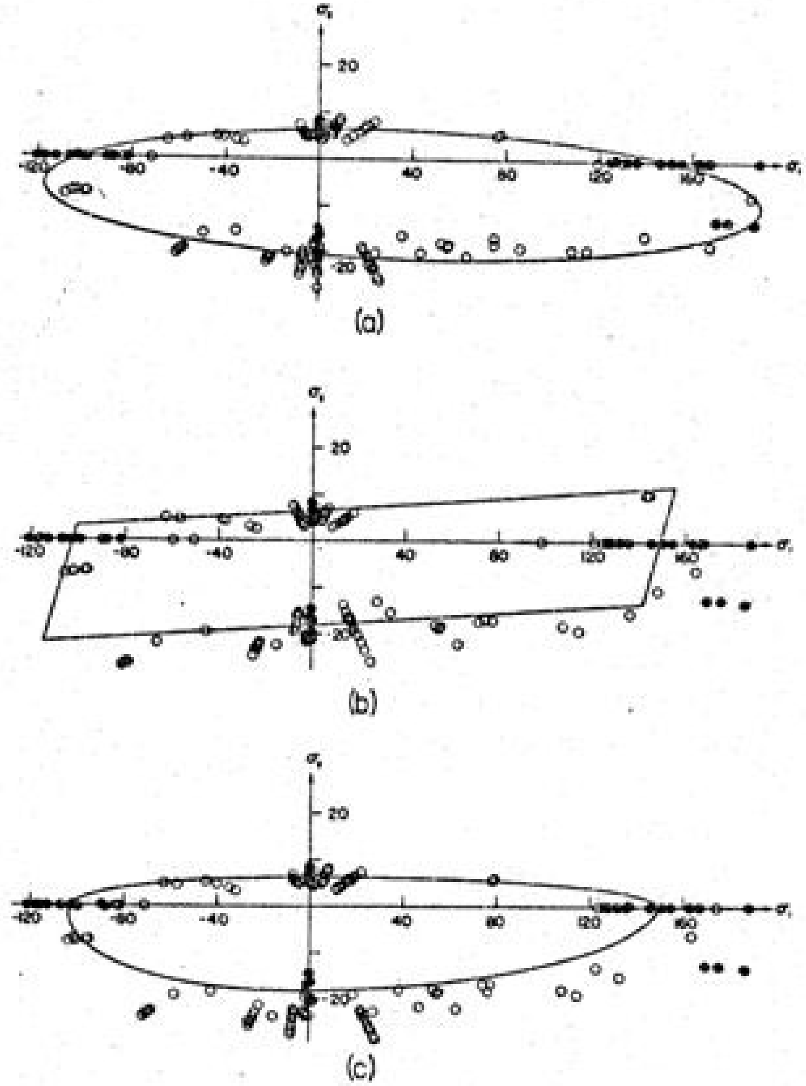


FIG. 18. Failure data convoluted onto $\sigma_1\sigma_2$ plane; stresses in {ksi}: (a) by tensor polynomial failure criterion; (b) by maximum strain failure criterion; (c) by modified von Mises–Hill failure criterion.

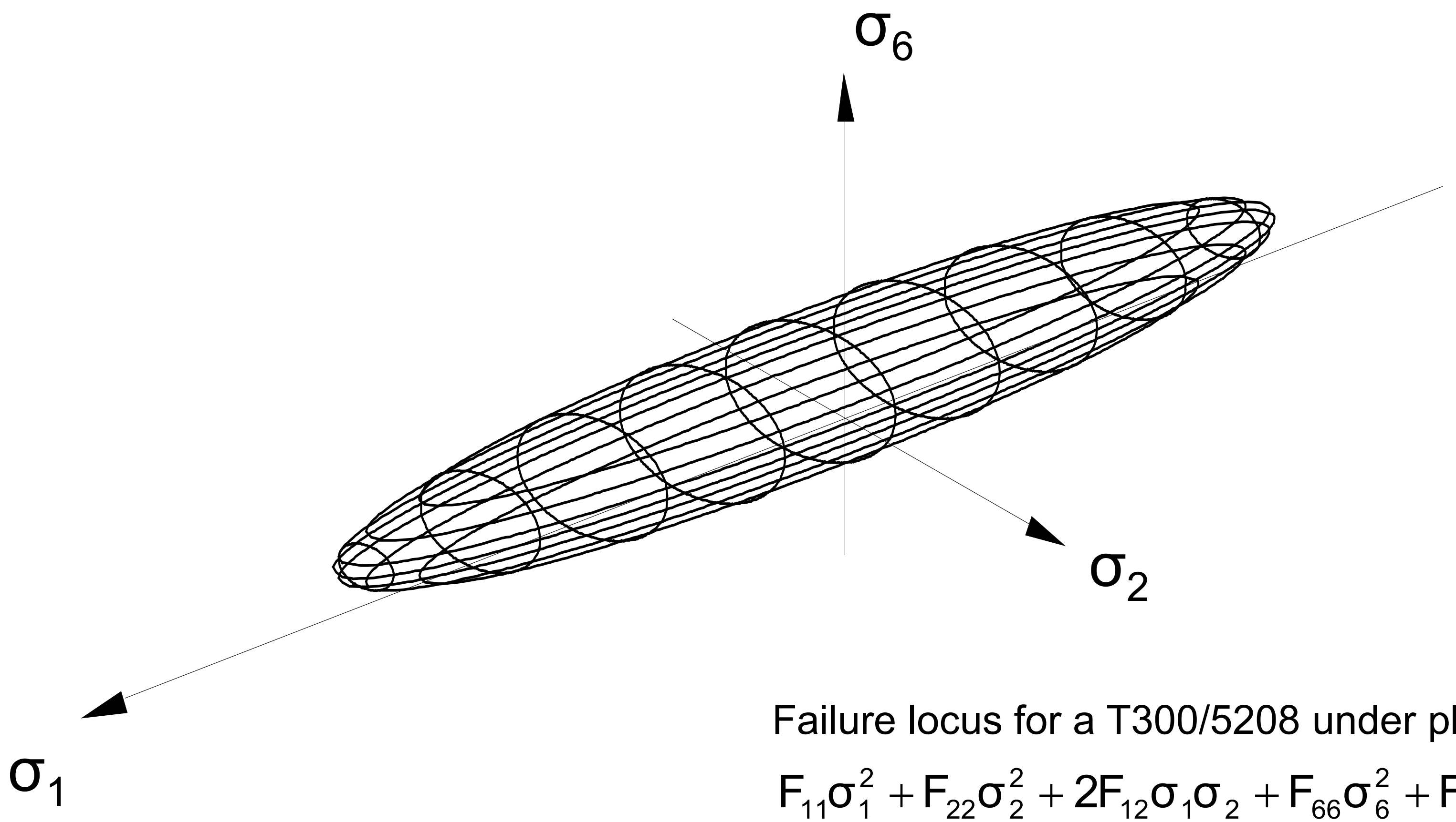
E. M. Wu: Phenomenological Anisotropic Failure Criterion, in COMPOSITE MATERIALS **2**, AP, Broutman, Krock Eds., 1974

Failure Loci

Failure criteria can assist material selection procedures

A failure locus offers a tool for the evaluation of failure criteria by comparing with available experimental data.

A failure locus, especially for anisotropic media, helps the designer to understand the material capacity for transferring loads – it helps in material selection at preliminary design stages



Failure locus for a T300/5208 under plane stress (1-2)

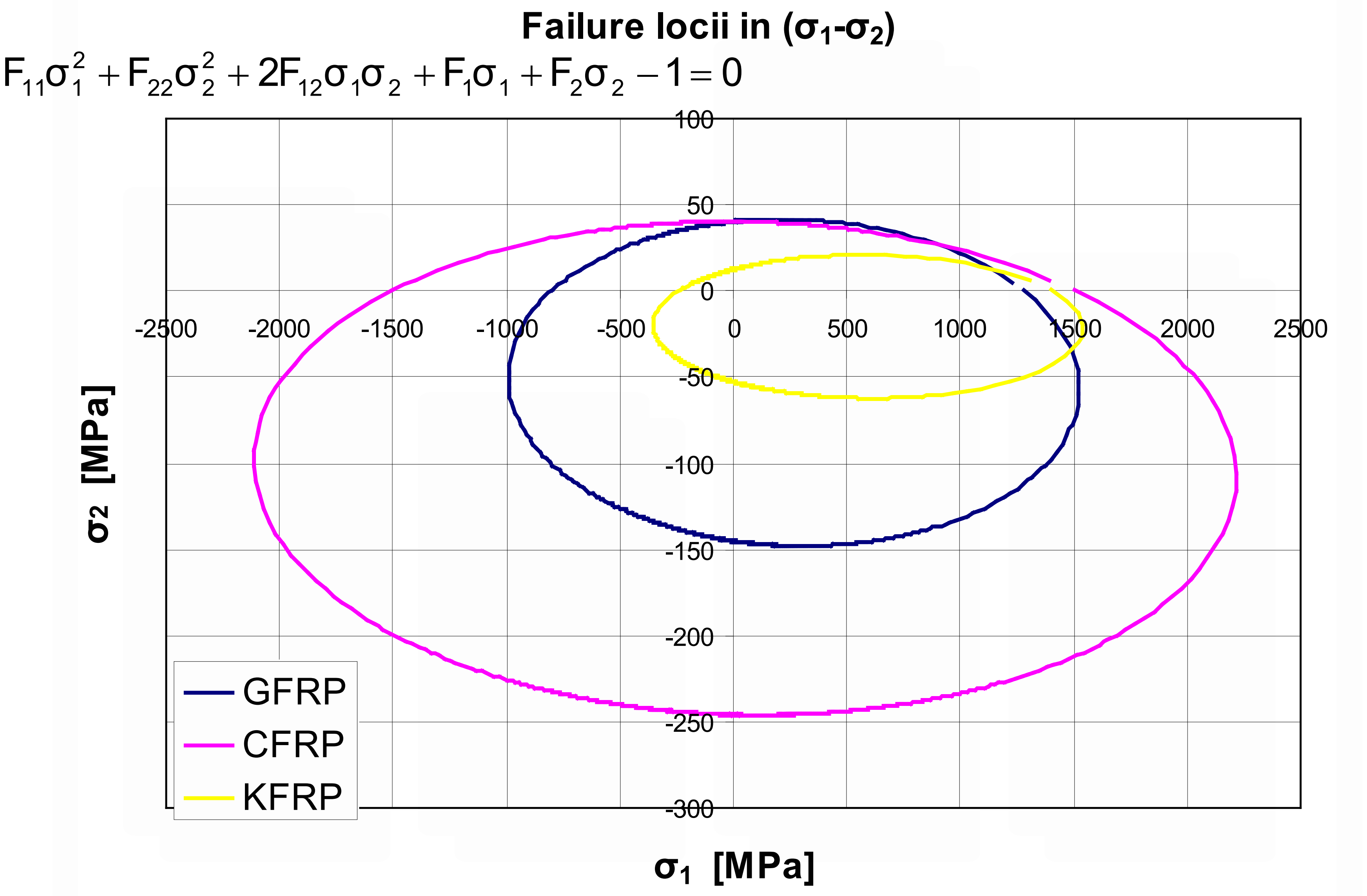
$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\sigma_6^2 + F_1\sigma_1 + F_2\sigma_2 - 1 = 0$$

For the transversely isotropic medium (UD) only 5 failure stresses should be determined: X, X´, Y, Y´, S. (the same for the orthotropic medium)

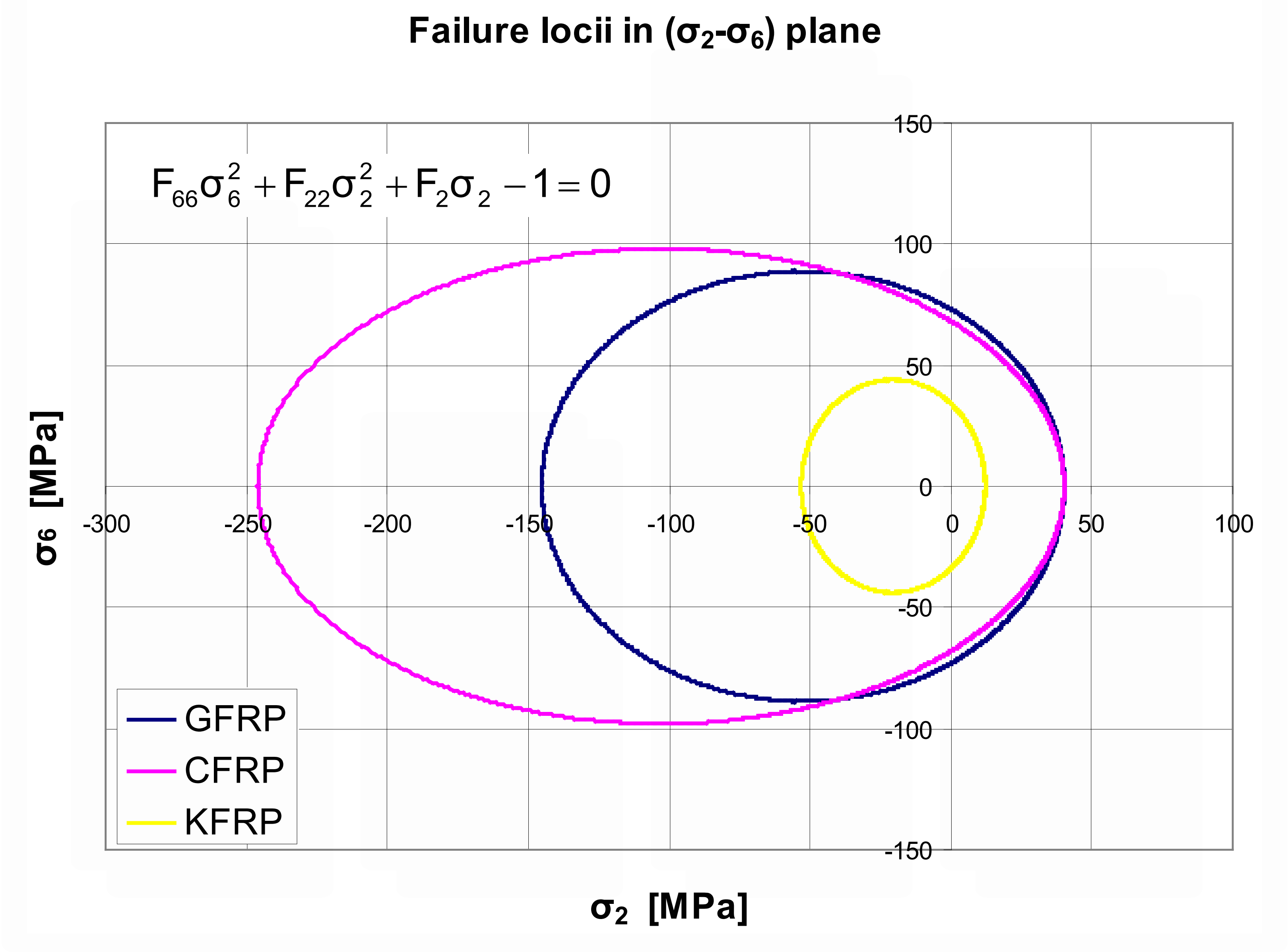
Failure stresses, in MPa, of (UD) composite materials

Commercial name	Material	X	X´	Y	Y´	S
T300/5208	Gr/Ep	1500	1500	40	246	68
B(4)/5505	B/Ep	1260	2500	61	202	67
AS/3501	Gr/Ep	1447	1447	52	206	93
Scotchply 1002 (3M)	Gl/Ep	1062	610	31	118	72
Kevlar 49	Ar/Ep	1400	235	12	53	34

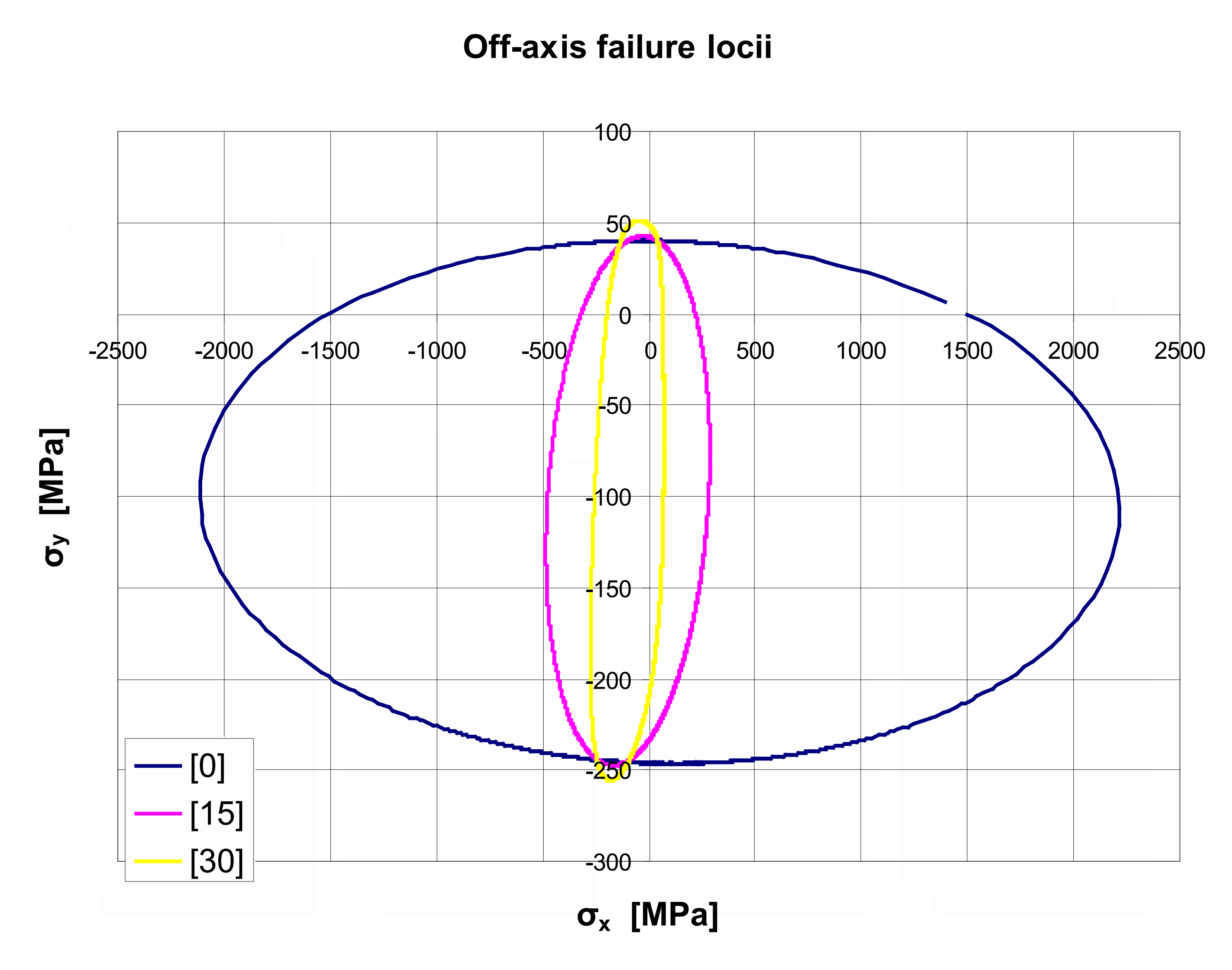
Representative failure locii for different composite materials ($F_{12}=-F_{11}/2$).



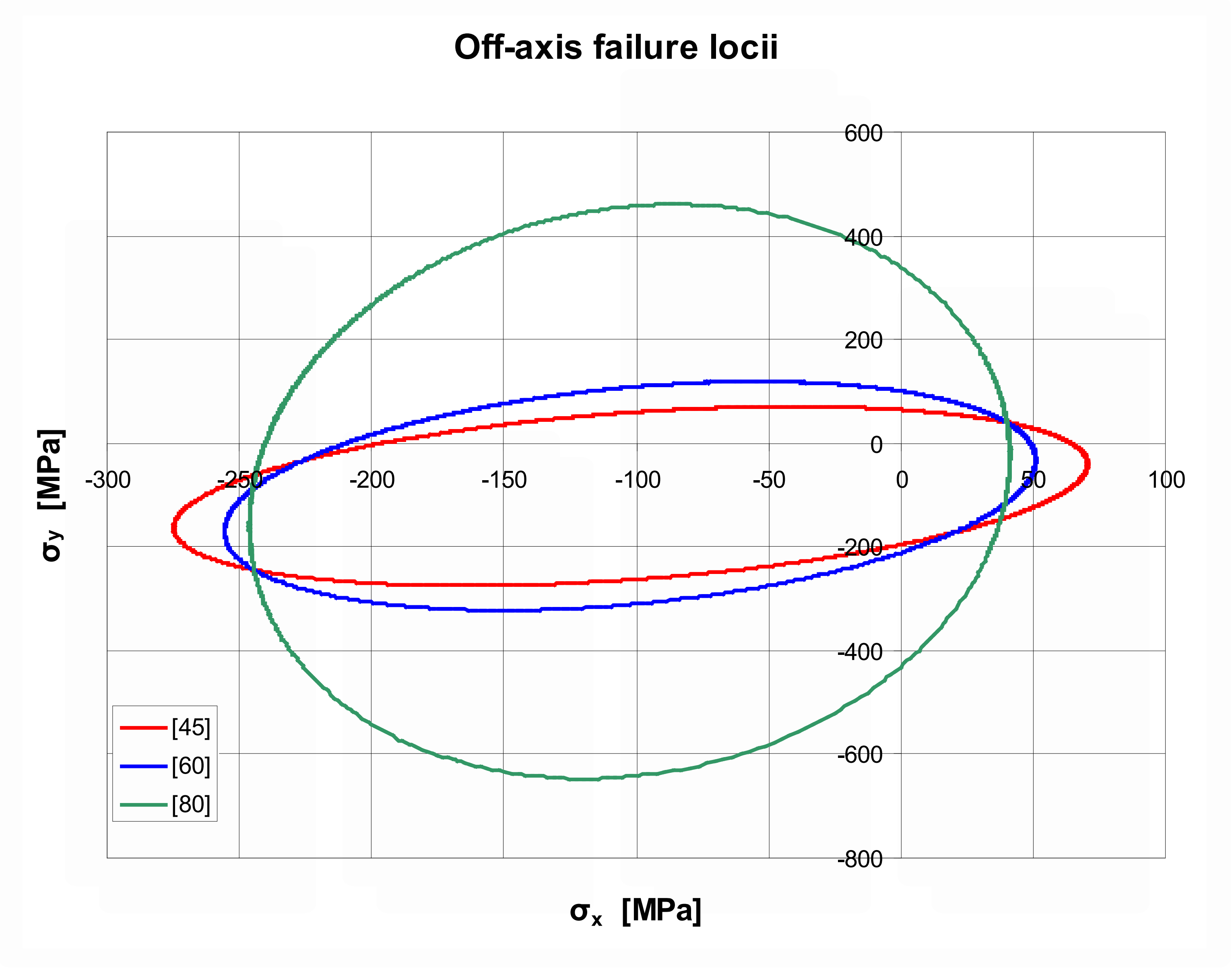
Representative failure locii for different composite materials



Failure locii for T300/5208 for different off-axis angles (1-2 plane)

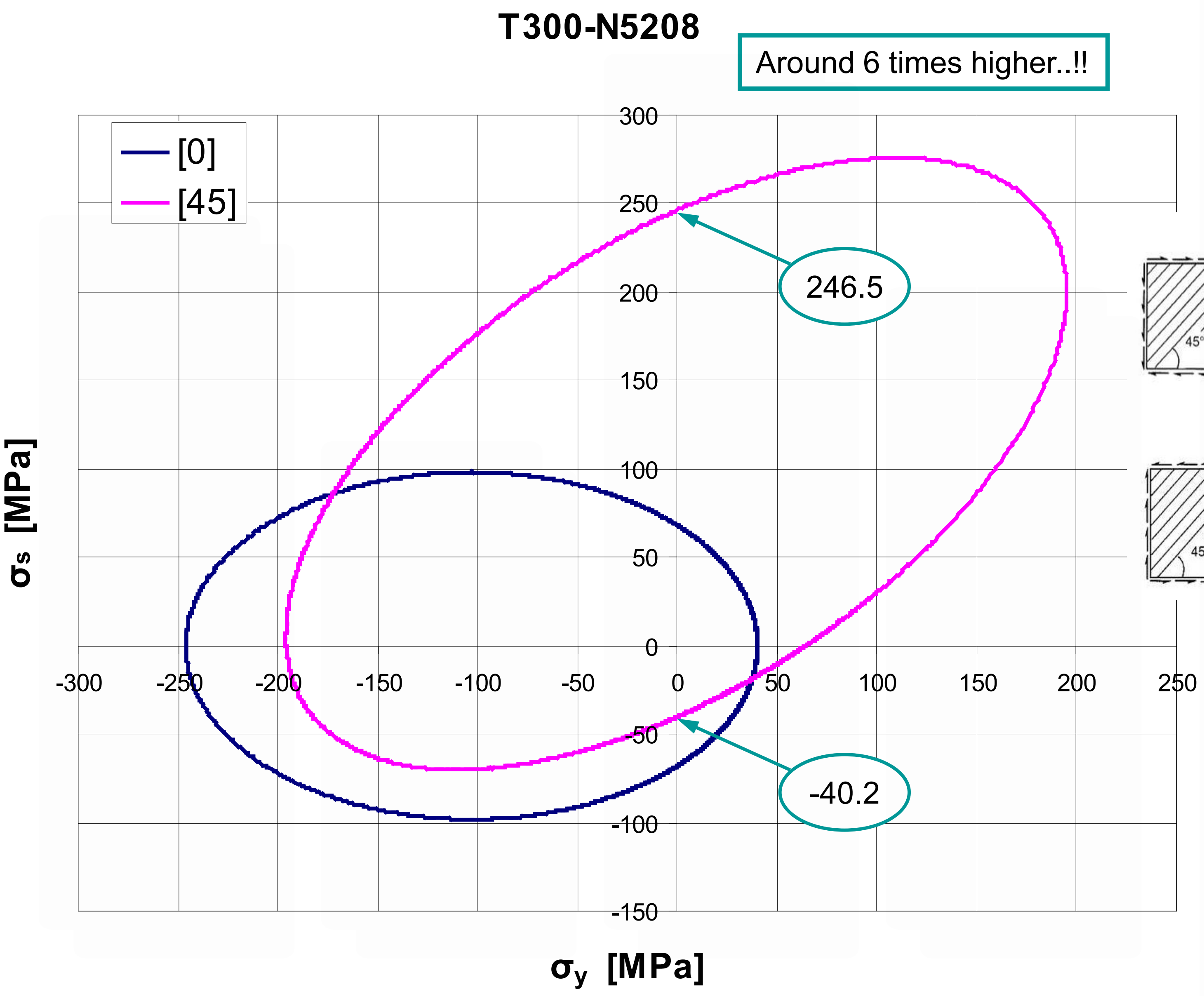


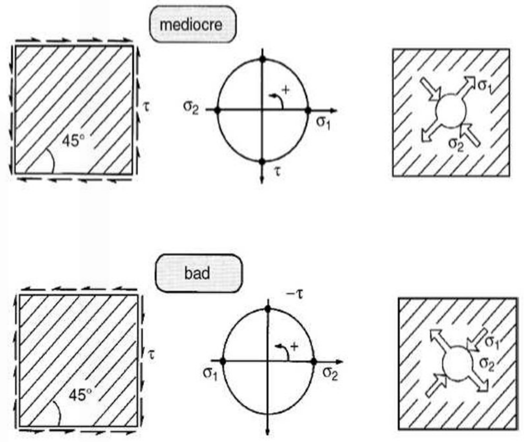
Failure locii for T300/5208 for different off-axis angles (1-2 plane)



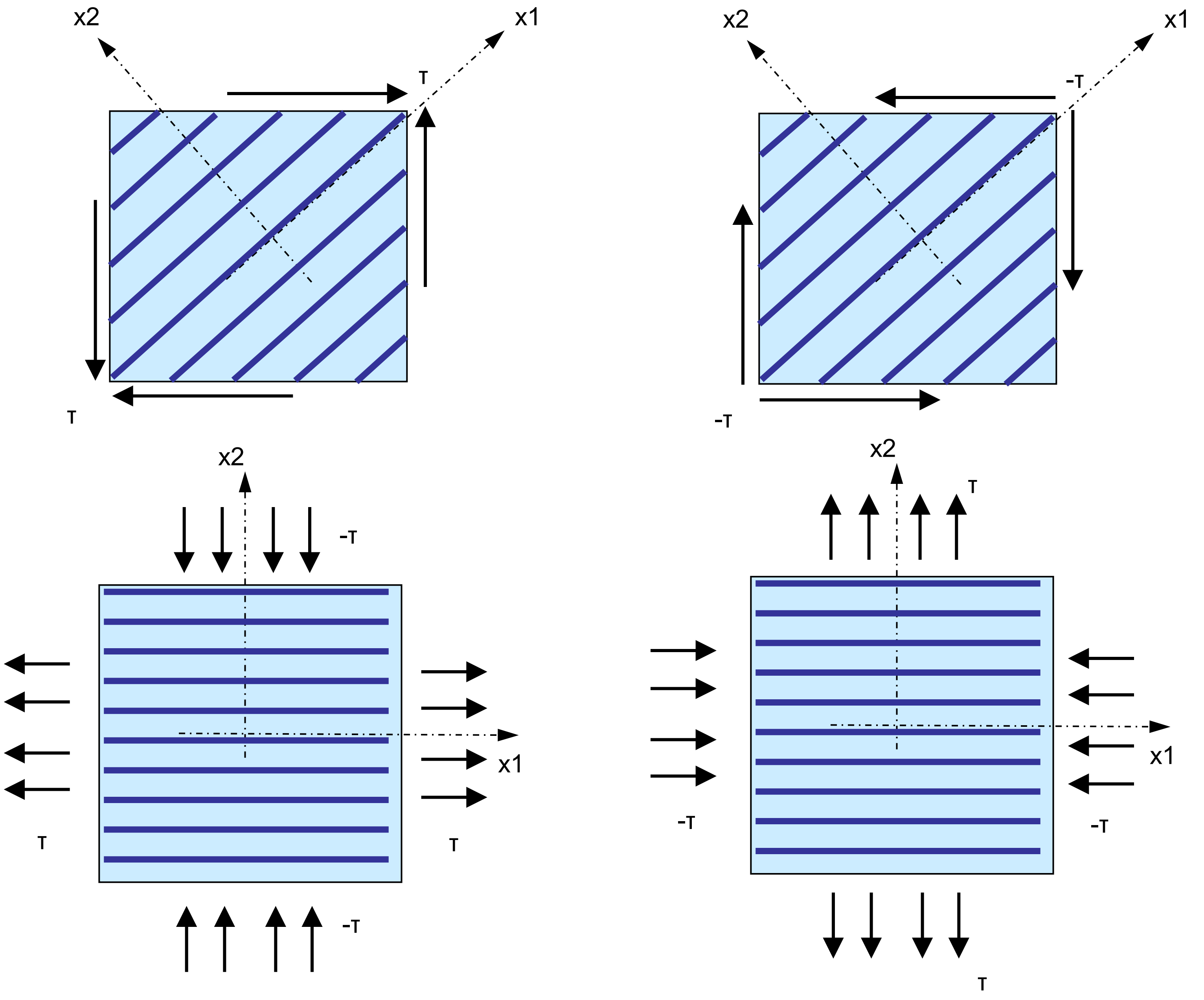
Failure locii for T300/5208 for different off-axis angles (2-6 plane)

Different "positive" and "negative" shear!

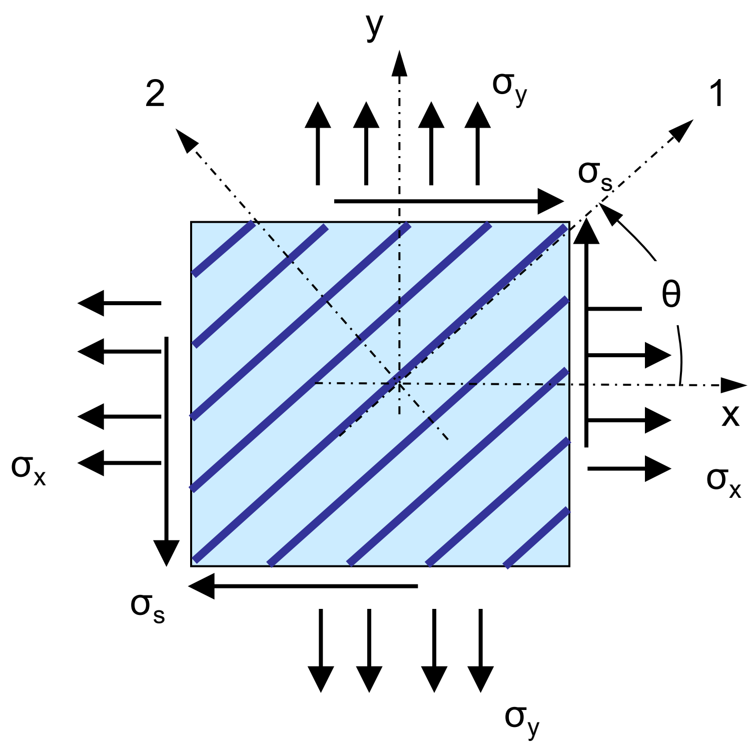




Equivalent stress states for on-axis co-ordinate and 45^{o} off-axis co-ordinate systems



Example I: **Determine the optimum fiber angle for maximum strength under a given plane stress state.**



A. Principal stress orientation and value:

$$\tan 2\theta_\sigma = \frac{2\sigma_s}{\sigma_x - \sigma_y}$$

$$\sigma_{I,II} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_s^2}$$

B. Minimizing the failure tensor polynomial:

$$f(\sigma_{ij}) = F_{ij}\sigma_i\sigma_j + F_i\sigma_i - 1 \leq 0 \quad i, j = x, y, s$$

The result is dependent on the selection of the failure criterion

Consider the next loading case of a laminate made of T300/N5208:

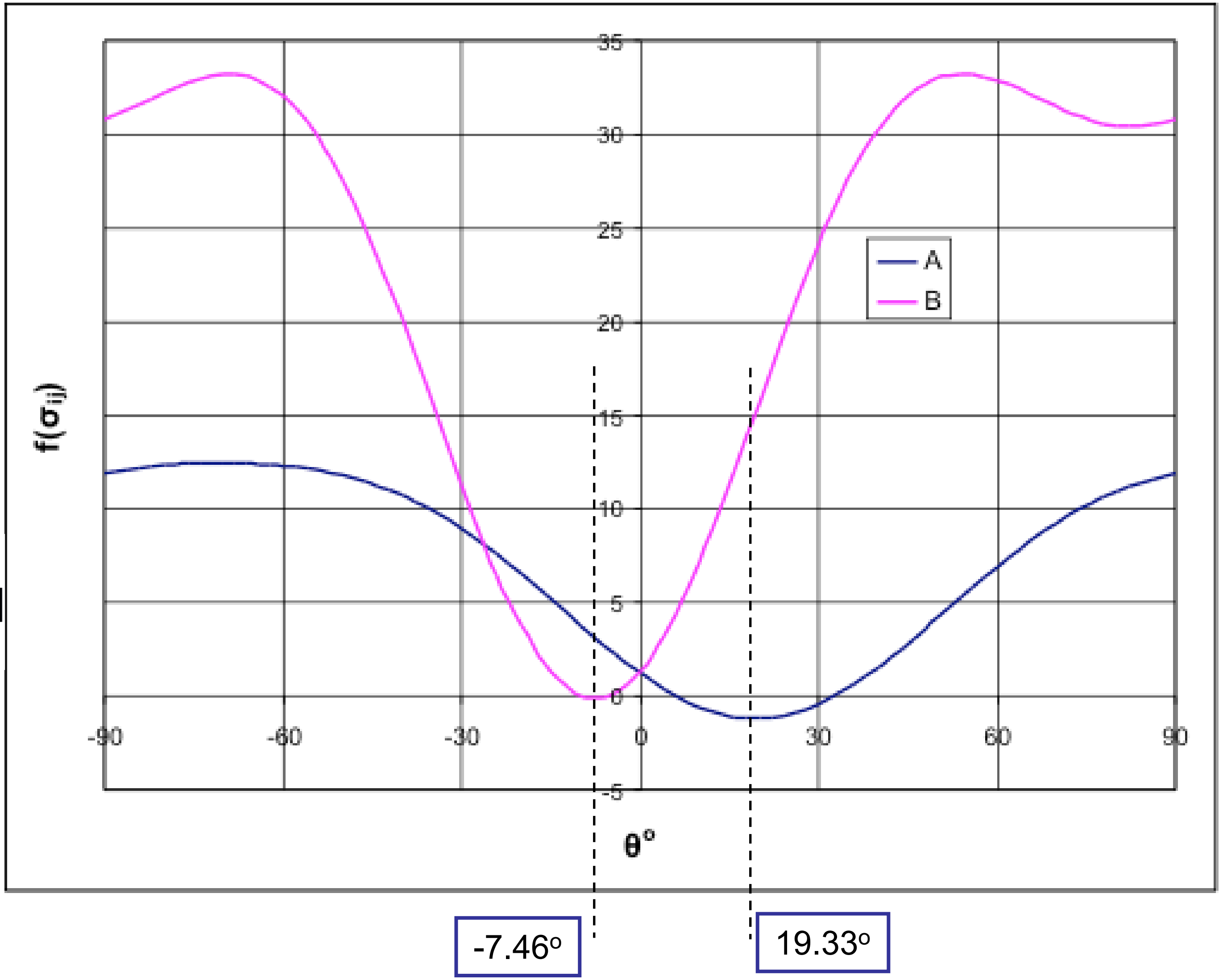
Loading case I: $\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} -450 \\ -225 \\ -90 \end{Bmatrix} [MPa] \rightarrow \begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} -193.43 \\ -481.57 \end{Bmatrix} [MPa]$ θ_σ=-70.67º

Fibers along an angle of 19.33º

Loading case II: $\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} 450 \\ -225 \\ -90 \end{Bmatrix} [MPa]$

$$\begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} 461.79 \\ -236.79 \end{Bmatrix} [MPa]$$

θ_σ=-7.46º



Coincidence for both criteria

Discussion

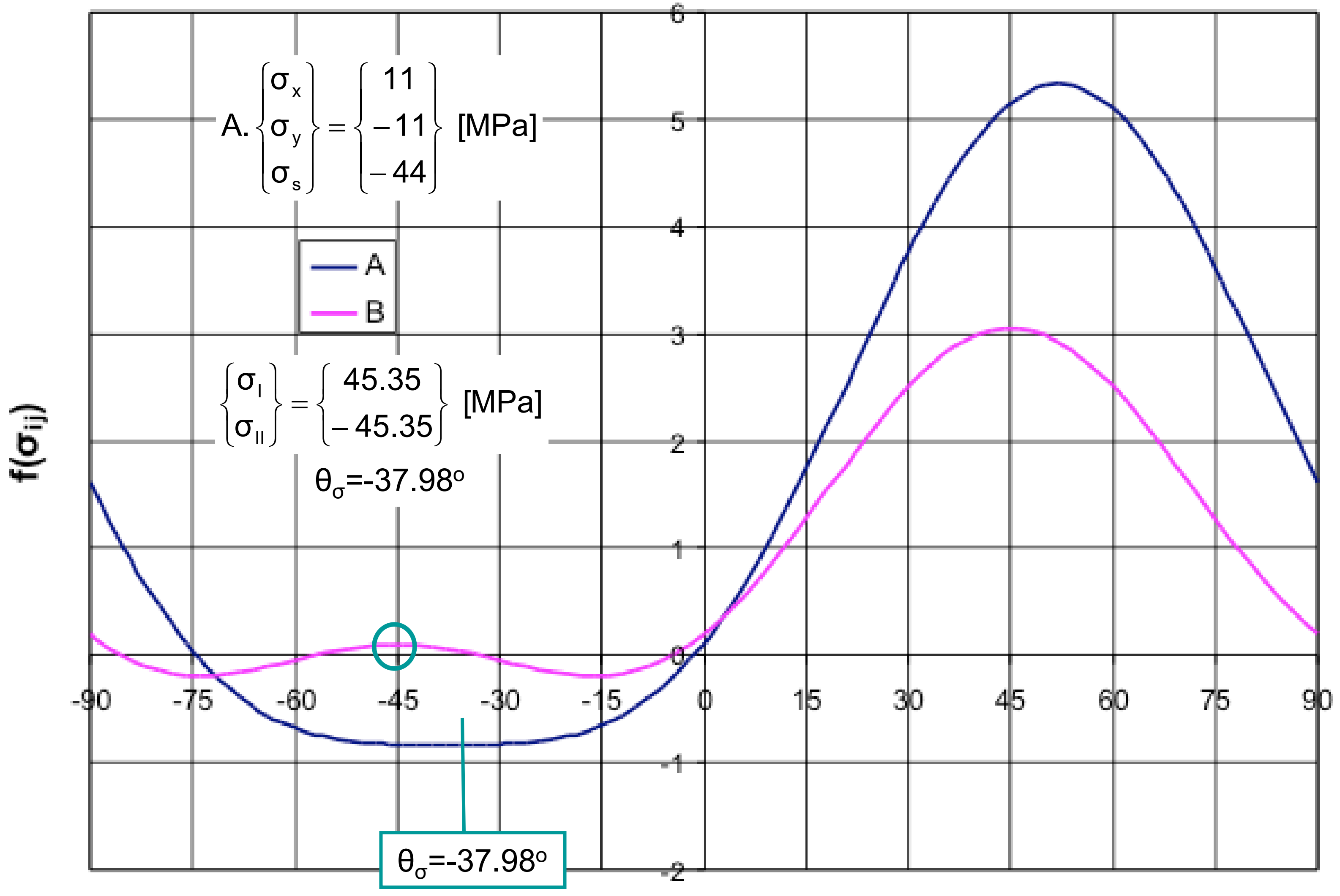
Previous results (for the given material and loading) verify the hypothesis that the fibers should be placed along the direction of the principal normal stresses.

However this is not the case for all materials and any loading case...

Consider the next loading cases for Kevlar 49:

$$A.\ \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = \begin{Bmatrix} 11 \\ -11 \\ -44 \end{Bmatrix} [MPa] \longrightarrow \begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \begin{Bmatrix} 45.35 \\ -45.35 \end{Bmatrix} [MPa] \quad \theta_\sigma = -37.98^o$$

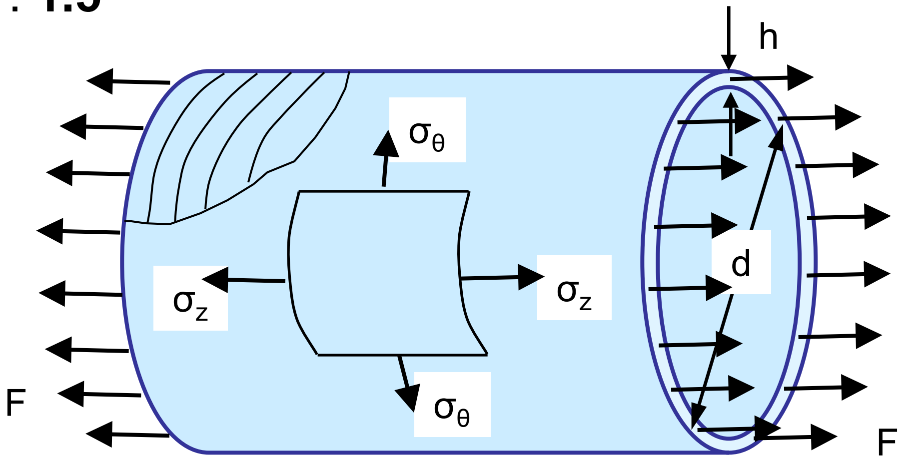
$$\text{B.}\ \begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_s\end{Bmatrix}=\begin{Bmatrix}-11\\ -11\\ -44\end{Bmatrix}\ [\text{MPa}] \longrightarrow \begin{Bmatrix}\sigma_I\\ \sigma_{II}\end{Bmatrix}=\begin{Bmatrix}33\\ -55\end{Bmatrix}\ [\text{MPa}] \quad \theta_\sigma=-45^o$$

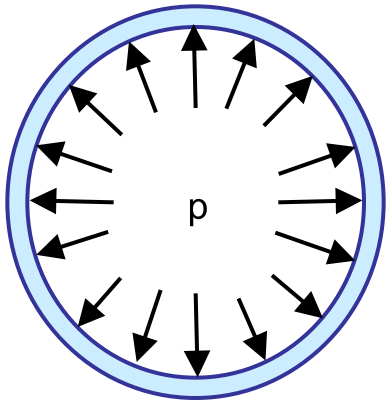


Different estimation from both criteria for a given loading

Example II: Define the minimum thickness of the walls of a tube with a Diameter *Φ*150 mm that would be fabricated by the filament winding technique.
The direction of the angles (winding angle) is the transverse to the longitudinal axis of the cylinder.

Loads: Internal pressure: **p=2 MPa**, Tensile force **F=20 kN**
Safety factor: **1.5**





$$\sigma_z = \frac{F}{\pi\, dh}$$

Plane stress

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_1\sigma_1 + F_2\sigma_2 - 1 = 0$$

$$\sigma_\theta = \frac{pd}{2h}$$

$$F_{11}\left(\frac{p'd}{2h}\right)^2 + F_{22}\left(\frac{F'}{\pi\, dh}\right)^2 + 2F_{12}\left(\frac{p'd}{2h}\right)\left(\frac{F'}{\pi\, dh}\right) + F_1\left(\frac{p'd}{2h}\right) + F_2\left(\frac{F'}{\pi\, dh}\right) - 1 \le 0$$

$$p' = 1.5p$$

$$F' = 1.5F$$

T300/N5208 $h \ge 1.603 \cdot 10^{-3} m$ → 9 layers of 200 μm each

γ_F=1.0 $h \ge 1.069 \cdot 10^{-3} m$ → 6 layers of 200 μm each

Project evaluation criteria

- **Topic and approach**
  - How do you transfer your problem from the complex structure to the infinitesimal element for the stress/strain analysis
  - Intermediate steps (not evaluated) – load estimation/simplifications
  - Intermediate steps (evaluated) – assumptions
  - MUST: include hand calculations of CLT
- **Report**
  - Concise and clear Correctness
  - Clarity of group involvement
- **Presentation**
  - Clarity – conveying the message without being chatty
  - Group performance